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Topic: Recurrence Relation and Compound Interest

Time: 45 mins   Marks: /45 marks

Calculator Assumed

Question One: [3, 8: 11 marks]

a)   Compare the amount of interest earned in one year on a $20 000 investment which is earning 8.8% interest p.a if interest is compounding:

   i)   annually

   ii)   monthly

   iii)   daily

b)   Write the recursive rule to determine the value of the investment at the end of each year for each of the different compounding periods mentioned in part a).

Question Two: [2, 4, 3, 2, 2: 13 marks]

Kerry is saving a deposit to buy a house. She has some money which she invests in an account paying 9.0% interest p.a compounding monthly. She adds $500 to this account at the end of each month.

Month (n)	Balance at the start of the month ($) ($T_n$)	Interest ($)	Installment ($)	Balance at the end of the month ($)
1	A	37.5	500	B
3	6079.03	45.59	500	6624.62
4	C	D	500	E

Kerry was trying to calculate her future savings but she spilled coffee on the table and some of the information became smudged.

a)   Calculate A, the value of the initial investment.

b)   Calculate the value of B, C D and E.

c)   Write the recursive rule to describe the amount of money in the account at the start of each month.

d)   What is the balance in the account at the end of the first year?

e)   If Kerry wants to have a $20 000 deposit in 18 months' time, how much do her monthly installments need to be?

Question Three: [8 marks]

Consider the following investment accounts for a $10 000 investment over 10 years.

A: 7.2% interest p.a compounding annually for the first 3 years and then monthly at a rate of 6% p.a there after.
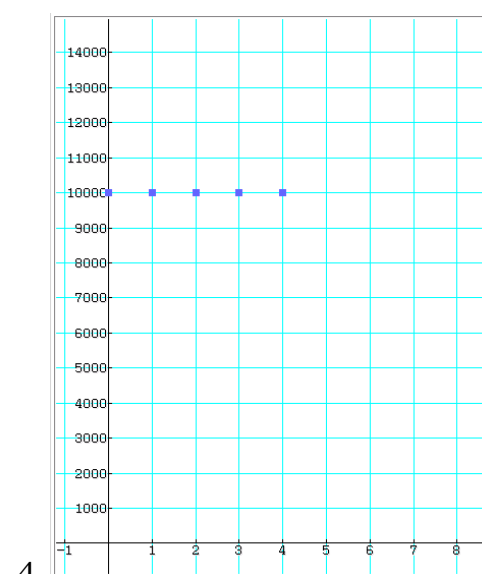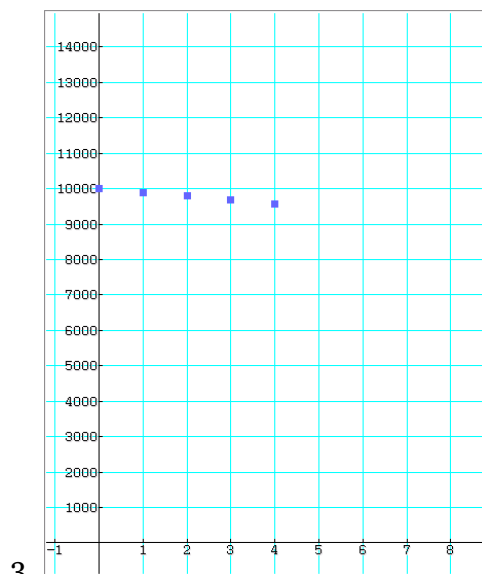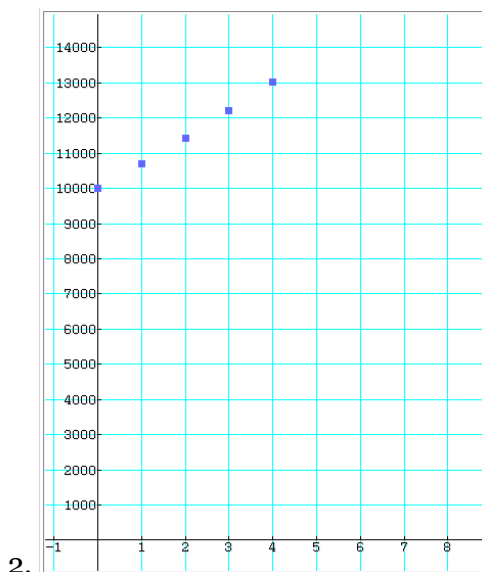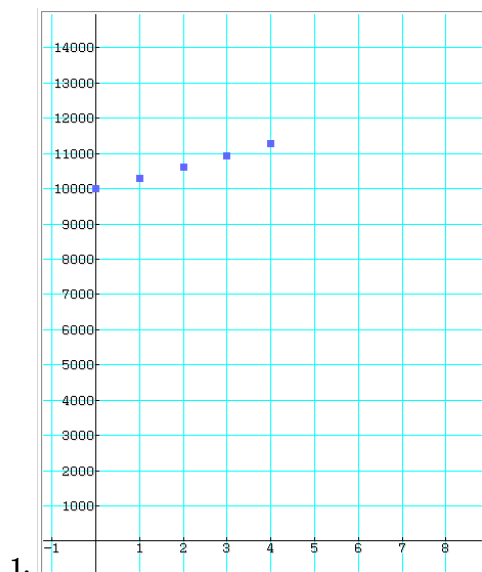
B: 5.0% interest p.a compounding monthly.

C: 4.5% interest p.a compounding daily.

Which account offers the highest return?

Question Four: [3 marks]

The following graphs relate to the recursive rules below. Match each graph to the relevant recursive rule.

1.

2.

3.

4.

$$A:\ T_{n+1} = 1.05T_n + 200\ T_0 = 10\ 000$$

$$B:\ T_{n+1} = 1.05T_n - 600\ T_0 = 10\ 000$$

$$C:\ T_{n+1} = 1.05T_n - 500\ T_0 = 10\ 000$$

$$D:\ T_{n+1} = 1.05T_n - 200\ T_0 = 10\ 000$$

Graph	1	2	3	4
Recursive Rule				

Question Five: [2, 3, 3, 2: 10 marks]

Jon Snow obtains a personal loan of $3 500 to buy a laptop. He is required to make $390 repayments at the end of each month. The table below shows the amount owing on the loan at the start of each month, the interest payable for that month, the repayment and the amount owing at the end of the month for the first few months of the loan.

Month	Amount owing at the start of the month ($)	Interest ($)	Repayment ($)	Amount owing at the end of the month ($)
1	3500	34.71	390	3144.71
2	3144.71	31.19	390	2785.89
3	2785.89	27.63	390	2423.52
4	2423.52	24.033	390	2057.55
5	2057.55	20.40	390	1687.96

a)   Calculate the annual interest rate.

b)   In which month would Jon Snow pay off the loan and how much is his final repayment.

c)   State the recursive rule to show the amount owing at the start of each month.

d)   What is the total amount of money Jon Snow pays for his laptop.

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Topic: Recurrence Relation and Compound Interest SOLUTIONS

Time: 45 mins   Marks: /45 marks

Calculator Assumed

Question One: [3, 8: 11 marks]

a)   Compare the amount of interest earned in one year on a $20 000 investment which is earning 8.8% interest p.a if interest is compounding:

   i)   annually

   Interest = $1760   ✓

   ii)   monthly

   Interest = $1832.75   ✓

   iii)   daily

   Interest = $1839.53   ✓

b)   Write the recursive rule to determine the value of the investment at the end of each year for each of the different compounding periods mentioned in part a).

i) $T_{n+1} = T_n \times 1.088 \; T_0 = 20\,000$
        ✓   ✓

ii) $T_{n+1} = T_n \times \left(1 + \frac{0.088}{12}\right)^{12} T_0 = 20\,000$
        ✓ ✓   ✓

iii) $T_{n+1} = T_n \times \left(1 + \frac{0.088}{365}\right)^{365} T_0 = 20\,000$
        ✓ ✓   ✓

Question Two: [2, 4, 3, 2, 2: 13 marks]

Kerry is saving a deposit to buy a house. She has some money which she invests in an account paying 9.0% interest p.a compounding monthly. She adds $500 to this account at the end of each month.

Month (n)	Balance at the start of the month ($) ($T_n$)	Interest ($)	Installment ($)	Balance at the end of the month ($)
1	A	37.5	500	B
3	6079.03	45.59	500	6624.62
4	C	D	500	E

Kerry was trying to calculate her future savings but she spilled coffee on the table and some of the information became smudged.

a)   Calculate A, the value of the initial investment.

$$A \times \frac{0.09}{12} = 37.5 \quad \checkmark$$

$$A = 5000 \quad \checkmark$$

b)   Calculate the value of B, C D and E.

$B = 5537.50 \qquad C = 6624.62 \qquad D = 49.68 \qquad E = 7174.30$

$\checkmark \qquad \checkmark \qquad \checkmark \qquad \checkmark$

c)   Write the recursive rule to describe the amount of money in the account at the start of each month.

$$T_{n+1} = T_n \times 1.0075 + 500 \quad T_1 = 5000$$

$\checkmark \quad \checkmark \quad \checkmark$

d)   What is the balance in the account at the end of the first year?

$11722.83 \checkmark\checkmark

e)   If Kerry wants to have a $20 000 deposit in 18 months' time, how much do her monthly installments need to be? $743.97 $\checkmark\checkmark$

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Question Three: [8 marks]

Consider the following investment accounts for a $10 000 investment over 10 years.

A: 7.2% interest p.a compounding annually for the first 3 years and then monthly at a rate of 6% p.a there after.

B: 5.0% interest p.a compounding monthly.

C: 4.5% interest p.a compounding daily.

Which account offers the highest return?

$A:After\ 3\ years\ \$12319.25\ and\ after\ 10\ years\ \$18729.82$

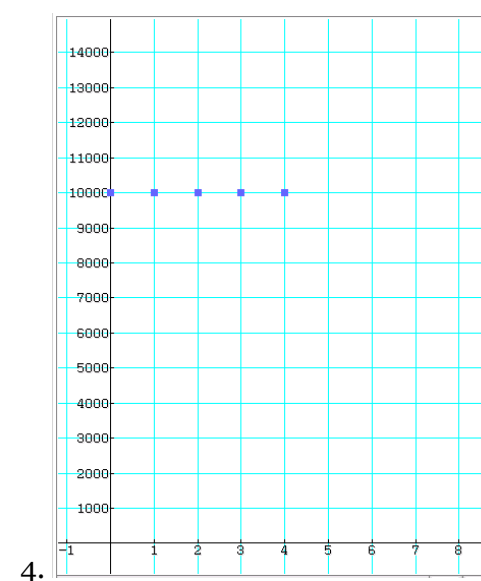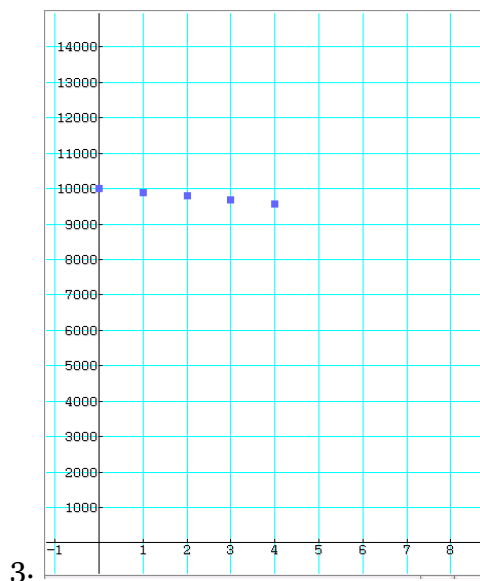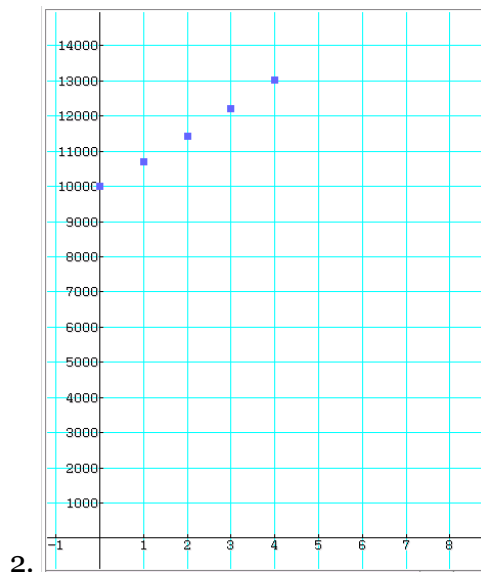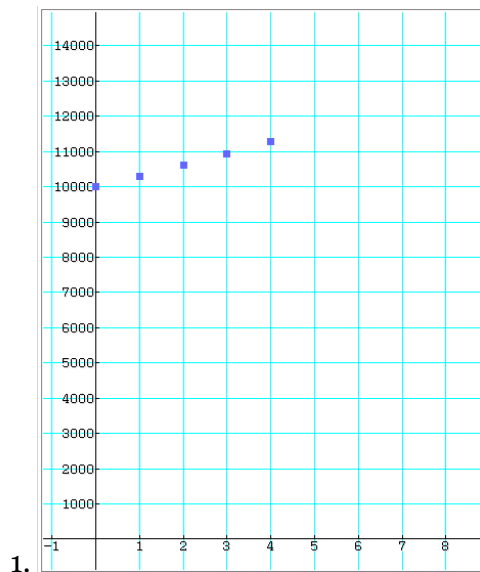$B:10\ 000 \times \left(1 + \dfrac{0.05}{12}\right)^{12 \times 10} = \$16470.09$

$C:\$15682.69$

$A:offers\ the\ highest\ return$

Question Four: [3 marks]

The following graphs relate to the recursive rules below. Match each graph to the relevant recursive rule.

1.

2.

3.

4.

$$A: \quad T_{n+1} = 1.05T_n + 200 \quad T_0 = 10\,000$$

$$B: \quad T_{n+1} = 1.05T_n - 600 \quad T_0 = 10\,000$$

$$C: \quad T_{n+1} = 1.05T_n - 500 \quad T_0 = 10\,000$$

$$D: \quad T_{n+1} = 1.05T_n - 200 \quad T_0 = 10\,000$$

Graph	1	2	3	4
Recursive Rule	D ✔	A ✔	B ✔	C

Question Five: [2, 3, 3, 2: 10 marks]

Jon Snow obtains a personal loan of \$3 500 to buy a laptop. He is required to make \$390 repayments at the end of each month. The table below shows the amount owing on the loan at the start of each month, the interest payable for that month, the repayment and the amount owing at the end of the month for the first few months of the loan.

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3	2785.89	27.63	390	2423.52
4	2423.52	24.033	390	2057.55
5	2057.55	20.40	390	1687.96

a)   Calculate the annual interest rate.

$11.9\%\ p.\,a$ ✔ ✔

b)   In which month would Jon Snow pay off the loan and how much is his final repayment.

Pays it off in 10 months. Final repayment is $390 - 215.73 = \$174.27$

✔   ✔   ✔

c)   State the recursive rule to show the amount owing at the start of each month.

$$T_{n+1} = T_n \times 1.00992 - 390 \quad T_1 = 3500$$

✔   ✔   ✔

d)   What is the total amount of money Jon Snow pays for his laptop.

$390 \times 9 + 174.27 = \$3684.27$

✔   ✔